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## Physics Letters B

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## Physical non-equivalence of the Jordan and Einstein frames

S. Capozziello<sup>a,b,\*</sup>, P. Martin-Moruno<sup>c</sup>, C. Rubano<sup>a,b</sup><sup>a</sup> Dipartimento di Scienze Fisiche, Università di Napoli "Federico II", Italy<sup>b</sup> INFN Sez. di Napoli, Compl. Univ. Monte S. Angelo, Ed.N, Via Cinthia, I-80126 Napoli, Italy<sup>c</sup> Colina de los Chopos, Instituto de Física Fundamental, Consejo Superior de Investigaciones Científicas, Serrano 121, 28006 Madrid, Spain

## ARTICLE INFO

## Article history:

Received 28 March 2010

Received in revised form 24 April 2010

Accepted 26 April 2010

Available online 27 April 2010

Editor: T. Yanagida

## Keywords:

Alternative theories of gravity

Cosmology

Conformal transformations

Noether symmetries

## ABSTRACT

We show, considering a specific  $f(R)$ -gravity model, that the Jordan frame and the Einstein frame could be physically non-equivalent, although they are connected by a conformal transformation which yields a mathematical equivalence. Calculations are performed analytically and this non-equivalence is shown in an unambiguous way. However this statement strictly depends on the considered physical quantities that have to be carefully selected.

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## 1. Introduction

The current accelerated expansion of the Universe, supported by a large number of observational data [1–3], is one of the most challenging issues of the modern physics. The assumption that Einstein's General Relativity (GR) is the correct theory of gravity leads to the consideration that approximately the 70% of the energy density of the Universe should be an unknown form of fluid called "dark energy", responsible of the mentioned acceleration. Even more, the largest part of the matter content is not constituted by standard baryonic matter, but by another unknown component called "cold dark matter" (CDM), constituting about the 25% of the total matter-energy budget. The most popular model able to describe this scenario is the  $\Lambda$ CDM, where it is considered that the dark energy component is simply the cosmological constant. Although  $\Lambda$ CDM model fits to a wide range of data [4], it is affected by strong theoretical shortcomings [5]. Specifically there is the cosmological constant problem [6], regarding the fact that the predicted value of the quantum-field vacuum energy density and the observed cosmological value are currently separated by 120 orders of magnitude, or the cosmic coincidence problem, which opens the question about why the today observed values of the CDM density and the cosmological constant energy density are of the same order of magnitude.

These shortcomings have motivated the study of a plethora of models which consider a dynamical dark energy characterized by an equation of state parameter  $w < -1/3$  ( $w = p/\rho$ ), recovering the  $\Lambda$ CDM model in the particular case that one has a constant parameter  $w = -1$ . Although these models could avoid in some cases the mentioned problems, [7], the origin of this fluid which produces anti-gravitational effects, violating at least one of the energy conditions [8], remains a mystery.

On the other hand, up to now, there is no definitive candidate for CDM, in spite of the efforts to identify its particle nature by its non-gravitational effects from space and ground-based experiments (comments in some experimental programs can be found in [9] and one of the main goals of the Large Hadron Collider at CERN is the identification of these particles [10]).

Since the validity on large astrophysical and cosmological scales of GR has never been tested, one could suppose that current observational datasets imply the non-validity of GR at those scales. Therefore, *Extended Theories of Gravity* (ETGs), which were initially introduced by quantum motivations, have been taken seriously in consideration. ETGs modify and enlarge the Einstein theory, adding into the effective action physically motivated higher order curvature invariants and/or non-minimally coupled scalar fields [11, 12].

Among ETGs,  $f(R)$ -theories are becoming of great interest, since they are the minimal extension of GR able to match the data without need of any dark energy or dark matter [13]. These theories modify the Einstein's action including a generic function  $f(R)$  of the Ricci scalar  $R$  instead of rigidly considering the Hilbert-Einstein action linear in  $R$  [14–17].

\* Corresponding author at: Dipartimento di Scienze Fisiche, Università di Napoli "Federico II", Italy.

E-mail address: [capozziello@na.infn.it](mailto:capozziello@na.infn.it) (S. Capozziello).

The Hilbert–Einstein action and the  $f(R)$ -action can be related by a conformal transformation [18–20], being the corresponding equations also connected by the same transformation. This fact shows that the Einstein frame and the Jordan frame are mathematically equivalent [21] but they could not be physically equivalent as pointed out in several papers (see e.g. [20,22,23]).

This is an old argument widely discussed in last decades (see e.g. [24] where a detailed discussion for dilaton gravity in two dimensions is reported). In [25], for example, the problem of physical non-equivalence of conformal frames has been considered: in that case the work done by a conformal transformation is capable of “creating” matter and so the two frames have not the same physical meaning (one is empty and another has matter). Anyway, this could mean that the conformal transformations change physics unlike the coordinate transformations. Besides, the method of the conformal transformation can be used to study the problem of energy-momentum content of the gravitational field using statefinders [26].

This is an open question that, up to now, has not been completely solved (see [27] for a review on the topic). In particular, a strong debate has been pursued about the Newtonian limit (i.e. small velocity and weak field) of fourth order gravity models. According to some authors, the Newtonian limit of  $f(R)$ -gravity is equivalent to the one of Brans–Dicke gravity with the Brans–Dicke parameter  $\omega = 0$ , so that the PPN parameters of these models turn out to be ill-defined. In a recent paper [28], this point has been carefully discussed considering that fourth order gravity models are dynamically equivalent to the O’Hanlon Lagrangian [29]. This is a special case of scalar-tensor gravity characterized only by self-interaction potential and that, in the Newtonian limit, this implies a non-standard behavior that cannot be compared with the usual PPN limit of GR. The result turns out to be completely different from the one of Brans–Dicke theory and, in particular, suggests that it is misleading to consider the PPN parameters of this theory with  $\omega = 0$  in order to characterize the homologous quantities of  $f(R)$ -gravity. In other words, this result can be considered an indication of the fact that conformally transformed theories could *not* be physically equivalent (see e.g. [28]). However, this statement has to be supported by the fact that methods to measure observable physical quantities should be completely independent of the frames or, at least, the relation of their observed values into the frames well established.

The aim of this work is to prove that the physical non-equivalence of Jordan frame and Einstein frame could be exactly demonstrated considering a suitable model and selecting physically reliable quantities. For this reason, we will take into account a  $f(R)$ -model which allows us to compare analytically the two frames showing the physical differences.

The layout of this Letter is the following. In Section 2, we review the  $f(R)$ -cosmological model presented in [30]. It is particularly interesting being exactly integrable and capable of describing dust matter (decelerated) phase and the following dark energy (accelerated) phase under the same standard. In Section 3, we perform the conformal transformation obtaining the mathematically equivalent model in the Einstein frame. The comparison of the model, in Jordan’s and Einstein’s frame, is presented in Section 4 showing the possible physical non-equivalence. Being the calculations completely analytical, the comparison can be performed in an unambiguous way. Finally, in Section 5, we summarize the results and draw our conclusions.

## 2. The model

A general action describing  $f(R)$ -gravity in four dimensions is

$$\mathcal{A} = \int d^4x \sqrt{-g} f(R) + \mathcal{A}_m, \quad (1)$$

where  $f(R)$  is a generic function of the Ricci scalar  $R$  and  $\mathcal{A}_m$  is the action of a perfect fluid minimally coupled with gravity. Obviously assuming  $f(R) = R$  the standard Einstein theory is recovered. Varying with respect to  $g_{\mu\nu}$ , we get the field equations

$$G_{\mu\nu} = T_{\mu\nu}^{curv} + \frac{T_{\mu\nu}^m}{2f'(R)}, \quad (2)$$

where

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \quad (3)$$

and  $T_{\mu\nu}^{curv}$  is an effective stress–energy tensor constructed by curvature terms in the following way

$$T_{\mu\nu}^{curv} = \frac{1}{f'(R)} \left\{ \frac{1}{2}g_{\mu\nu}[f(R) - Rf'(R)] + f'(R)_{;\mu\nu} - g_{\mu\nu}f'(R)_{;\alpha}{}^{\alpha} \right\}. \quad (4)$$

This tensor is zero for  $f(R) = R$ . The prime indicates derivatives with respect to  $R$ .

In a Friedmann–Robertson–Walker (FRW) metric, taking into account a dust-matter perfect fluid, a point-like Lagrangian can be obtained

$$\mathcal{L} = a^3[f(R) - f'(R)R] + 6a^2f''(R)\dot{R}\dot{a} + 6f'(R)a\dot{a}^2 - 6kf'(R)a + D, \quad (5)$$

where  $D$  represents the standard amount of dust fluid, such that  $\rho = D/a^3$  [31]. The energy function  $E_{\mathcal{L}}$ , corresponding to the  $\{0, 0\}$ -Einstein equation, is

$$E_{\mathcal{L}} = 6f''(R)a^2\dot{R}\dot{a} + 6f'(R)a\dot{a}^2 - a^3[f(R) - f'(R)R] + 6kf'(R)a - D = 0. \quad (6)$$

The equations of motion for  $a$  and  $R$  are respectively

$$f''(R) \left[ R + 6H^2 + 6\frac{\ddot{a}}{a} + 6\frac{k}{a^2} \right] = 0, \quad (7)$$

$$6f'''(R)\dot{R}\dot{a} + 6f''(R)\ddot{R} + 6f'(R)H^2 + 12f'(R)\frac{\ddot{a}}{a} = 3[f(R) - f'(R)R] - 12f''(R)H\dot{R} - 6f'(R)\frac{k}{a^2}, \quad (8)$$

where  $H \equiv \dot{a}/a$  is the Hubble parameter. Eq. (7) ensures the consistency, since  $R$  coincides with the definition of the Ricci scalar in the FRW metric.

The choice<sup>1</sup>  $f(R) = -|R|^{3/2}$  in Eqs. (1)–(8) produces a theory able to describe dust matter and dark energy combined phases in a FRW spacetime, without the need of any extra field introduced *ad hoc* (see [30,32] for details). In this particular case, the point-like FRW Lagrangian (5) is

$$\mathcal{L} = \frac{a^3}{2}|R|^{3/2} - \frac{9}{2}a^2|R|^{-1/2}\dot{R}\dot{a} + 9|R|^{1/2}a\dot{a}^2 - 9k|R|^{1/2}a + D, \quad (9)$$

and the energy function

<sup>1</sup> The reason for the absolute value stays only in the fact that, with our conventions,  $R$  turns out to be negative. It is obviously possible to rewrite everything with  $f(R) = R^{3/2}$  and  $R > 0$ .

$$\begin{aligned}
E_{\mathcal{L}} &= -\frac{9}{2}a^2|R|^{-1/2}\dot{R}\dot{a} + 9|R|^{1/2}a\dot{a}^2 \\
&\quad - \frac{a^3}{2}|R|^{3/2} + 9k|R|^{1/2}a - D \\
&= 0.
\end{aligned} \tag{10}$$

Referring to [30], it is possible to show that such a model has a Noether symmetry that allows to find out an exact solution for Eqs. (6), (7) and (8) for this particular  $f(R)$ , that is

$$a(t) = \sqrt{a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t} \tag{11}$$

with

$$\begin{aligned}
a_4 &= \frac{\Sigma_1^2}{144}, & a_3 &= \frac{\Sigma_1 \Sigma_0}{36}, & a_2 &= \frac{\Sigma_0^2}{24} - k, \\
a_1 &= \frac{\Sigma_0^3}{36\Sigma_1} - 2k \frac{\Sigma_0}{\Sigma_1} + \frac{4D}{9\Sigma_1},
\end{aligned} \tag{12}$$

where  $k$  is the spatial curvature,  $\Sigma_1$  the Noether charge and  $\Sigma_0$  the integration constant.

In order to fix the coefficients  $a_i$ 's, we have to consider time units in which the current time is  $t_0 = 1$ . However, one can construct the dimensionless quantity  $H_0 t_0 \sim 0.93$  which has to remain constant. Therefore the Hubble parameter results of order one (we choose  $H_0 = 1$  for simplicity). The current deceleration parameter can also be fixed taking  $q_0 = -0.4$ , which could describe a reasonable current acceleration. Finally, a unit value for the present scale factor value is considered. This assumption can be always done if no restriction on the value of  $k$  is imposed. In order to fix the remaining free parameters, we consider  $a_4 = 0.106$ , which leads  $\Omega_{m0} = 0.0418032$  (with  $\Omega_m = \rho/[6H^2 f'(R)]$ ), very close to the expected content of baryonic matter. With these assumptions, the scale factor is

$$a(t) = \sqrt{\frac{t}{5}[2 + 0.53(t-1)^3 + t + 2t^2]} \tag{13}$$

and the Ricci scalar

$$R(t) = \frac{9(41 + 212t)^2}{212t(147 + 259t + 41t^2 + 53t^3)}. \tag{14}$$

This model describes a spatially open Universe,  $k \simeq -0.5$ . We have to note that the measurable quantity is not this parameter but  $\Omega_{k0} \simeq 0.02$  which is very small. Moreover, since the requirement  $\Omega_k \simeq 0$  is derived by the spectrum of the CMBR data, and these data strongly depend on the standard  $\Lambda$ CDM model, we cannot conclude that this feature is needed in our  $f(R)$ -model.

In fact, this solution, in principle, seems to reproduce satisfactorily observational data, out from the trivial fulfillment of the *a priori* fixed. In particular, the scale factor (13) is able to emulate a dust dominated epoch necessary for the structure formation, with only a difference with respect the standard  $a_F \sim t^{2/3}$  of the 3% in the range  $2 \leq z \leq 4$ , and the distance modulus derived by this model is also able to reproduce the SNeIa data [30].<sup>2</sup>

### 3. Conformal transformation

Let us consider now the gravitational part of our action, i.e.

$$\mathcal{A}_G = - \int d^4x \sqrt{-g} |R|^{3/2}, \tag{15}$$

which, by defining a auxiliary scalar field  $\varphi$  in the following way,

$$\varphi(R) = \sqrt{\frac{3}{2}} \ln(3|R|^{1/2}), \tag{16}$$

can be written as

$$\mathcal{A}_G = \int d^4x \sqrt{-g} \left[ -\frac{|R|}{2} e^{\sqrt{2/3}\varphi} + \frac{1}{54} e^{3\sqrt{2/3}\varphi} \right]. \tag{17}$$

The new field  $\varphi$  does not introduce any physical new feature, since it is only a way to recast the further gravitational degrees of freedom related to  $f(R)$ -gravity. In fact, it can be seen that this is the case, since the  $\varphi$ -field equation obtained from Eq. (17) produces only Eq. (16). If we perform a conformal transformation by the conformal parameter

$$b(t) = \exp\left(\frac{\varphi}{2} \sqrt{\frac{2}{3}}\right), \tag{18}$$

which is a function of the time  $t$  since  $\varphi(R(t)) = \varphi(t)$ , the resulting action is the Hilbert–Einstein action with a scalar field  $\varphi(t)$

$$\mathcal{A}_G = \int d^4x \sqrt{-\tilde{g}} \left[ -\frac{|\tilde{R}|}{2} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + V(\varphi) \right], \tag{19}$$

where  $\|\tilde{g}_{\mu\nu}\| = b(t)^2 \text{diag}(-1, a(t)^2, a(t)^2, a(t)^2)$ ,  $\tilde{R}$  is the Ricci scalar of the metric  $\tilde{g}_{\mu\nu}$  and  $V(\varphi) = \exp[\sqrt{2/3}\varphi]/54$ . If we define a new time variable  $\tau$ , in such a way that  $d\tau = b(t)dt$ , we recover a FRW metric  $\tilde{g}_{\mu\nu}$ , but now with a scale factor  $a_E(\tau) = b(\tau)a(\tau)$

$$\mathcal{A}_G = \int d^4\tilde{x} \sqrt{-\tilde{g}} \left[ -\frac{|\tilde{R}|}{2} - \frac{1}{2} \tilde{g}^{\mu\nu} \tilde{\partial}_\mu \tilde{\varphi} \tilde{\partial}_\nu \tilde{\varphi} + \tilde{V}(\tilde{\varphi}) \right]. \tag{20}$$

$\tilde{R}$  is the Ricci scalar of the metric  $\tilde{g}_{\mu\nu}$ ,  $\tilde{R}(\tau) = \tilde{R}(t(\tau))$ ,  $\tilde{\varphi}(\tau) = \varphi(t(\tau))$  and  $\tilde{V}(\tilde{\varphi}) = V(\varphi)$ . Taking also into account the mentioned transformations in the matter component, we obtain the total action in the Einstein frame and the point-like FRW Lagrangian

$$\begin{aligned}
\mathcal{L} &= 3a_E(\partial_\tau a_E)^2 - 3ka_E - \frac{a_E^3}{2}(\partial_\tau \tilde{\varphi})^2 \\
&\quad + a_E^3 \tilde{V}(\tilde{\varphi}) + e^{-\tilde{\varphi}/\sqrt{6}} \tilde{\rho}_m,
\end{aligned} \tag{21}$$

where  $\tilde{\rho}_m = D/a_E^3$ . Such a Lagrangian shows a coupling between the matter term and the scalar field, which will produce the non-conservation of both fluids individually.

The Einstein equations yield

$$\tilde{G}_{\mu\nu} = \tilde{T}_{\mu\nu}^{\tilde{\varphi}} + \tilde{T}_{\mu\nu}^m + \tilde{T}_{\mu\nu}^{int}, \tag{22}$$

where

$$\tilde{T}_{\mu\nu}^{\tilde{\varphi}} = \tilde{\partial}_\mu \tilde{\varphi} \tilde{\partial}_\nu \tilde{\varphi} - \frac{1}{2} \tilde{\partial}_\alpha \tilde{\varphi} \tilde{\partial}^\alpha \tilde{\varphi} \tilde{g}_{\mu\nu} + \tilde{V}(\tilde{\varphi}) \tilde{g}_{\mu\nu}, \tag{23}$$

$$\tilde{T}_{\mu\nu}^m = \text{diag}(\tilde{\rho}_m, 0, 0, 0), \tag{24}$$

and

$$\tilde{T}_{\mu\nu}^{int} = (e^{-\tilde{\varphi}/\sqrt{6}} - 1) \text{diag}(\tilde{\rho}_m, 0, 0, 0). \tag{25}$$

It should be noted that, whereas  $\tilde{T}_{\mu\nu}^m$  is conserved  $\tilde{T}_{\mu\nu}^{\tilde{\varphi}}$  and  $\tilde{T}_{\mu\nu}^{int}$  do not fulfill any conservation law separately, but  $(\tilde{T}_{\mu\nu}^{\tilde{\varphi}} + \tilde{T}_{\mu\nu}^{int})^{;\mu} = 0$ . This result has to be taken into account in order to compare results in Jordan and Einstein frames.

<sup>2</sup> This choice of the parameters is interesting because it produces results which turn out to be reasonably good at least from the point of view of observational tests. However, the following comparison with the Einstein frame is not dependent on this choice.

#### 4. Jordan frame versus Einstein frame

In the previous section, we have shown how to perform a conformal transformation of  $f(R)$ -gravity to obtain GR with a dynamical scalar field, being therefore both frames mathematically equivalent. However, this mathematical equivalence does not necessarily ensure the physical equivalence of both frames. In fact, whereas, in the Jordan frame, the matter term is not coupled to any field or to gravity, in the Einstein frame there is a coupling between the matter and the scalar field, appearing as an interaction term in the Einstein equations (22). This fact is crucial in comparing the physics in the two systems.

In order to show that the two frames could be physically equivalent, we have to compare the physical quantities of the mentioned two frames. This is a delicate issue since the selection of such quantities should be unambiguous.

Through the definition of the conformal factor, Eq. (18), and Eqs. (14) and (16), one finds the explicit form of this parameter in terms of  $t$

$$b(t) = \frac{3\sqrt{41 + 212t}}{\sqrt{106(147t + 259t^2 + 41t^3 + 53t^4)^{1/4}}}, \quad (26)$$

with  $t$  the cosmic time in the Jordan frame, which is related to the cosmic time in the Einstein frame

$$\tau = \int b(t) dt. \quad (27)$$

Since  $a_E(t) = b(t)a(t)$ , Eq. (26) allows to obtain the scale factor in the Einstein frame in terms of  $t$  and, therefore, in terms of  $\tau$  through Eq. (27). In such a way, taking into account Eqs. (18) and (27), one can know, in principle, the explicit form of  $\tilde{\varphi}(\tau)$ . Unfortunately, it is not possible to obtain an analytic solution for  $\tau(t)$ , but we can perform a complete analytic study in terms of  $t$ , noting that, in the Einstein frame, it is only an arbitrary parameter and not the cosmic time. We thus maintain the dot for derivation with respect to  $t$  and write explicitly the derivatives w.r.t. the cosmic time  $\tau$ . This procedure will not affect the final results, because they will be set in terms of the redshift, which is an observable quantity.

Taking into account that  $a_E(t) = b(t)a(t)$ , we get the Hubble parameter in the Einstein frame

$$H_E(t) = \frac{\partial_\tau a_E}{a_E} = \frac{1}{b(t)} \frac{\dot{a}_E}{a_E}, \quad (28)$$

and a deceleration factor

$$q_E(t) = -\frac{(\partial_\tau^2 a_E) a_E}{(\partial_\tau a_E)^2} = -\frac{\ddot{a}_E a_E}{\dot{a}_E^2} + \frac{\dot{b} a_E}{b \dot{a}_E}. \quad (29)$$

Since the redshift can also be defined in terms of the parameter  $t$ ,

$$z_E(t) = -1 + \frac{a_{E,0}}{a_E(t)}, \quad (30)$$

where  $a_{E,0}$  is the current scale factor, we can eliminate the (unphysical) parameter  $t$ , by considering couples of parametric equations. In order to perform this study, we must fit  $t_0 = t(\tau_0)$ , and we do that demanding that the dimensionless parameter  $q_{E,0} = -0.4$  as it was required in the Jordan frame, setting the value  $t_0 \simeq 1.24$ . Figs. 1 and 2 show that the Hubble parameter  $H(z)$  and the deceleration parameter  $q(z)$ , respectively, are different in the Jordan and Einstein frames. This means that the frames are not physically equivalent (in fact, it would be enough that one of these physical functions were different in the two frames).

One can also compare the dimensionless quantity  $\Omega_{m,0}$  in both frames. In the Jordan frame, one can easily see, from the

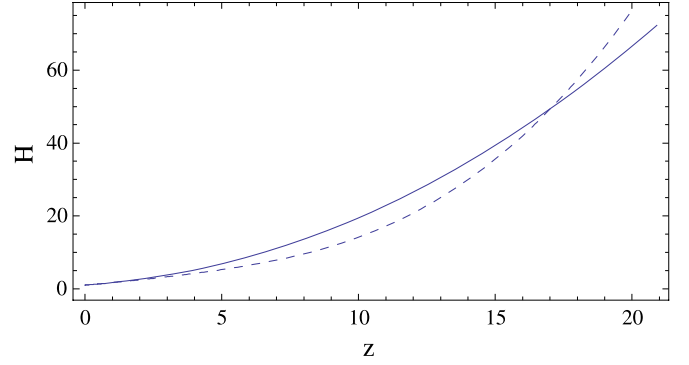


Fig. 1. Comparison of the Hubble parameter,  $H(z)$  in the Jordan frame and in the Einstein frame (dashed line), where the Hubble parameter in the Einstein frame has been normalized with its current value.

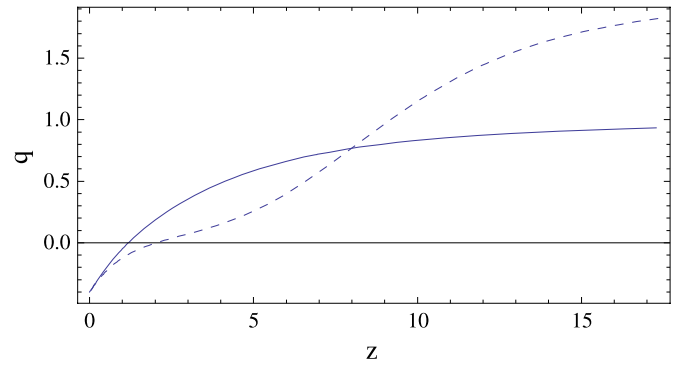


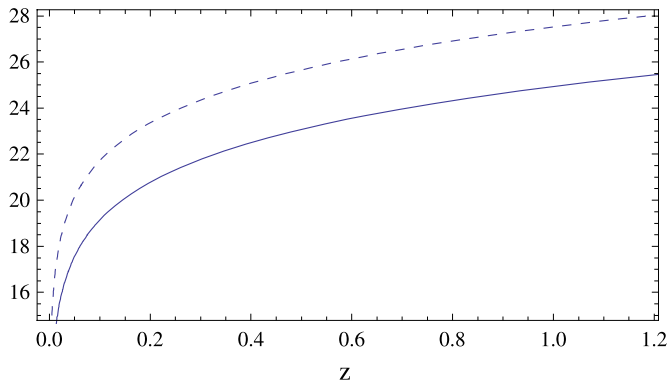
Fig. 2. Comparison of the deceleration parameter,  $q(z)$  in the Jordan frame and in the Einstein frame (dashed line).

00-component of Eq. (2), that it must be defined as  $\Omega_{m,0} = \rho_{m,0}/(6f'(R)H_0^2)$  and takes a value compatible with the baryonic component of the Universe, i.e., around 0.04. This parameter is defined in the Einstein frame as  $\tilde{\Omega}_{m,0} = \tilde{\rho}_{m,0}/(3H_{E,0}^2)$ , and takes a value which is more than twice the value in the Einstein frame, that is  $\tilde{\Omega}_{m,0} \simeq 0.09$ . On the other hand, in the Einstein frame there is an interaction term which produces  $\tilde{\Omega}_{int,0} = (1/b - 1)\tilde{\rho}_{m,0}/(3H_{E,0}^2) = -0.0567$ , therefore its absolute value is more than one half the value of the matter component, so it should produce some observable effect.

In order to show even more clearly than the Jordan and Einstein frames are not equivalent, we illustrate this fact in the following way. Let us consider two different researchers studying the model presented in Section 2 following two different routes. One of them refers all its calculations to the original Jordan frame and conclude that this model can describe the distance modulus data, as it is shown in [30]. The other one considers that the Jordan frame and the Einstein frame are physically equivalent and calculate also the distance modulus, but in the Einstein frame. As it is shown in Fig. 3, they obtain different functions. Since the function calculated in the Jordan frame fits the mentioned data, while the function obtained in the Einstein frame does not, the second research would conclude that the model does not describe our Universe, whereas the first one would continue with his study.

#### 5. Conclusions

In this Letter, we have shown that the Jordan and Einstein frames could not be physically equivalent according to the choice of observable quantities. We have considered a particular  $f(R)$ -model and the resulting model in the Einstein frame, obtained by a



**Fig. 3.** Comparison of the distance modulus in the Jordan frame and in the Einstein frame (dashed line).

conformal transformation. The discrepancy between these models has clearly been shown in the coupling term between the matter component and the scalar field which appears in the conformally transformed model in the Einstein frame, and in Figs. 1, 2 and 3, which prevent that the two models could describe the “same” Universe. These differences cannot be considered as a mistake coming from any numerical approximation, since all the study is performed in an analytic way.

On the other hand, conformal transformations between the Jordan and Einstein frames result extremely useful if used in a consistent way. Thus, since the Jordan and Einstein frame are mathematically equivalent, one can perform the calculations in the more convenient frame whenever one conformally transforms the obtained functions to the “true frame”.

The identification of the “true” physical frame is a controversial question. But if one consider that the Jordan (Einstein) frame is the true frame, one must refer all results to this frame in order to compare them with the observational data. One can also take an equitable position and consider that the “true frame” is that which is in agreement with the observational data to a larger extent. This point remain still open, although the model presented here and in [30] is able to fulfill some observational tests (without the introduction of any dark stuff) and to reproduce a dust matter decelerated phase, before the current accelerated one, may help us to find an answer in a future. Obviously a deeper study of the mentioned model is still necessary and the physical non-equivalence between frames should be tested also for other models and observables.

## Acknowledgements

P.M.M. gratefully acknowledges the financial support provided by the I3P framework of CSIC and the European Social Fund.

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